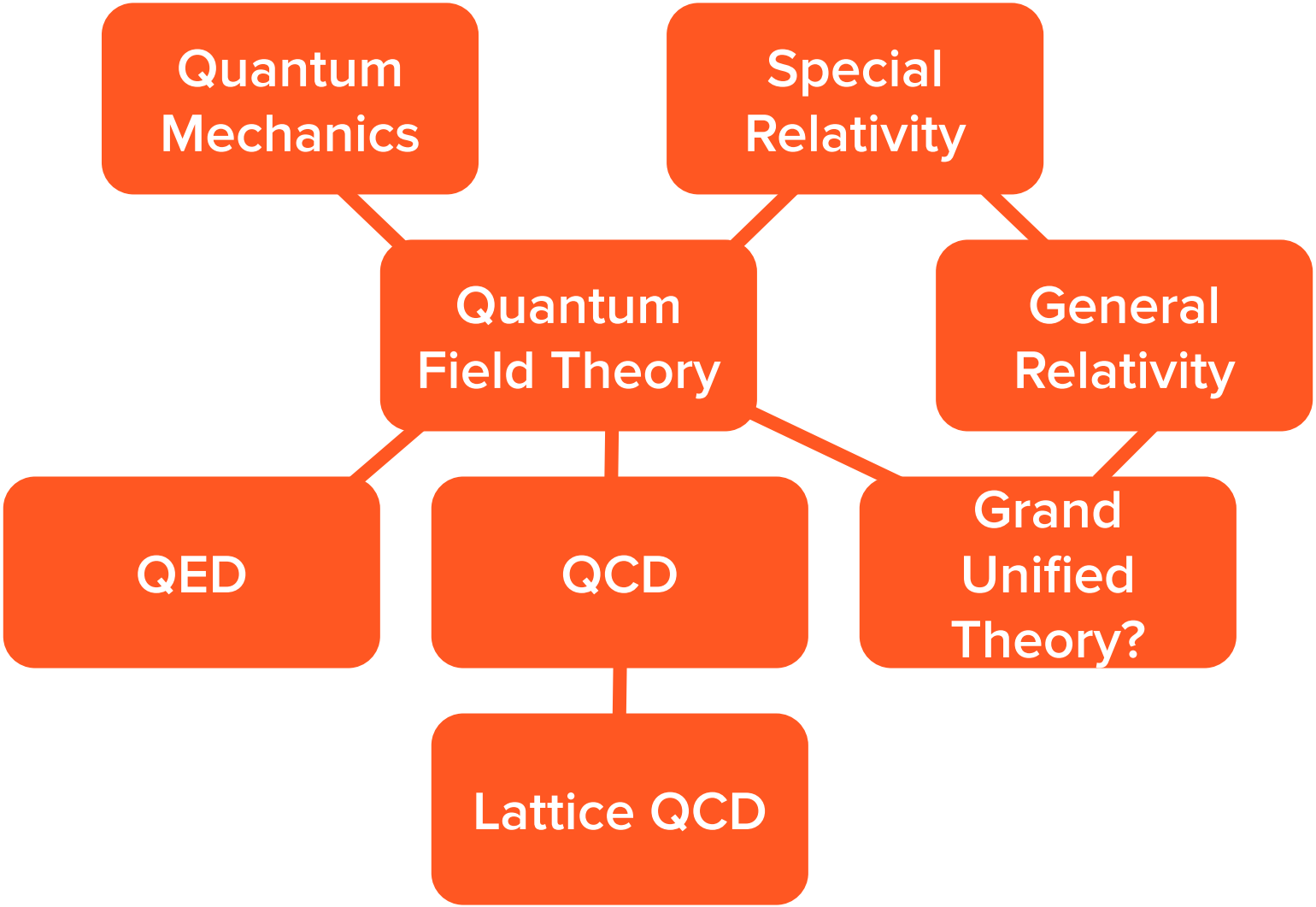
Lattice Quantum ChromoDynamics

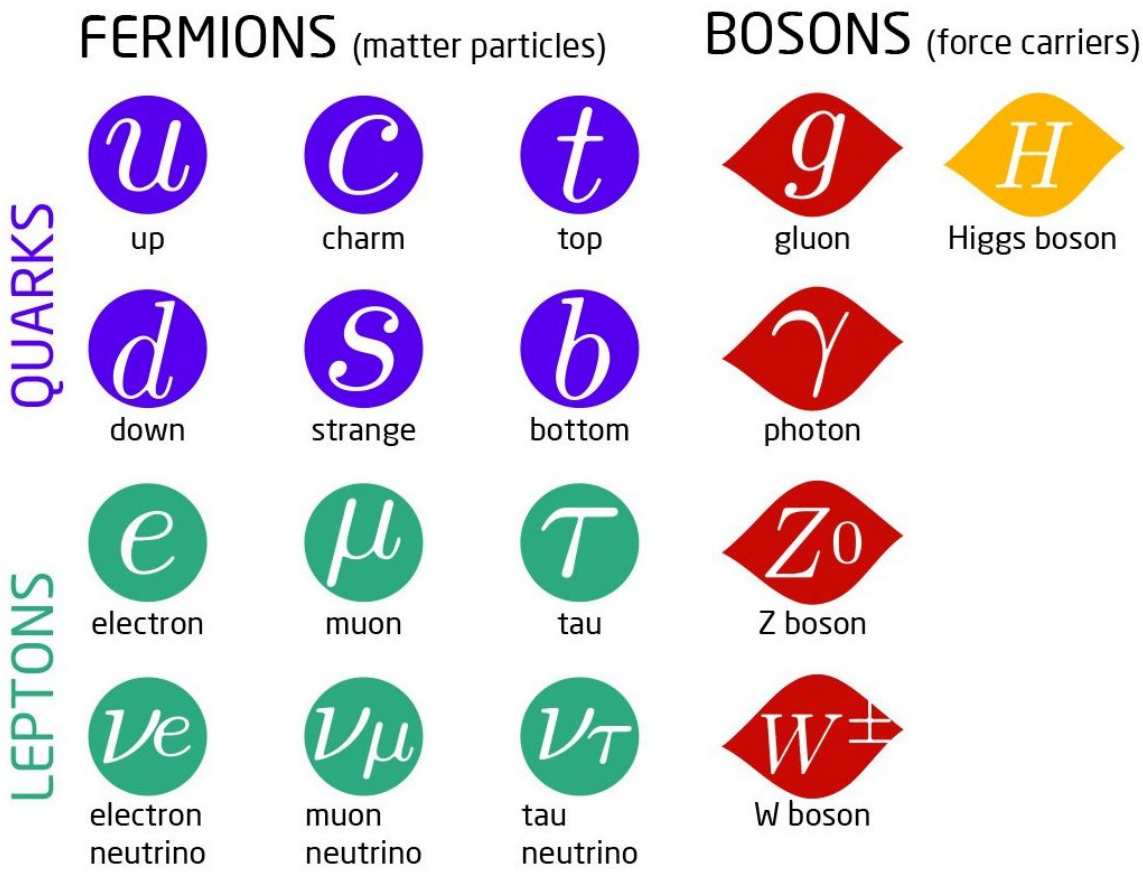
Ed van Bruggen


Quantum Mechanics
Special Relativity
Quantum Field Theory
General Relativity
QED
QCD
Grand Unified Theory?
Lattice QCD

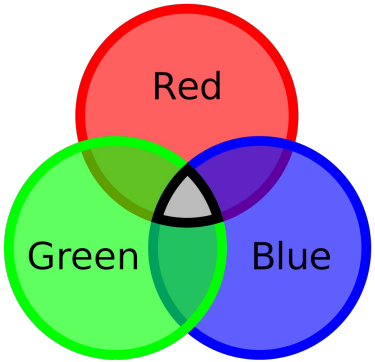
Four Fundamental Forces

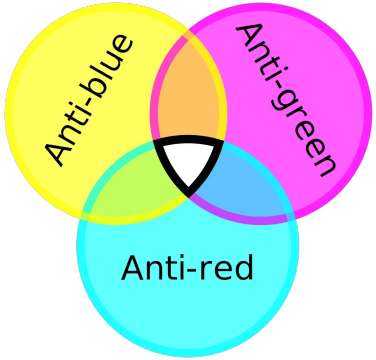
	Relative Strength	Potential	Acts on	Exchange Particle
Strong Nuclear	1	~r	Color	Gluon ***g***
Electromagnetism	1/137	~1/r	Charge	Photon **γ**
Weak Nuclear	10^{-6}	$\sim e^{-r}$	Fermions	$\mathbf{W^+}$ $\mathbf{W^-}$ $\mathbf{Z^0}$
Gravity	10^{-41}	~1/r	Mass	Graviton??

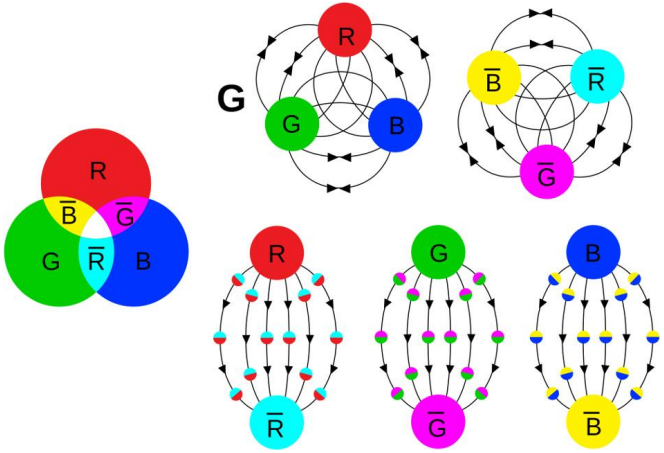
The Standard Model of Particle Physics



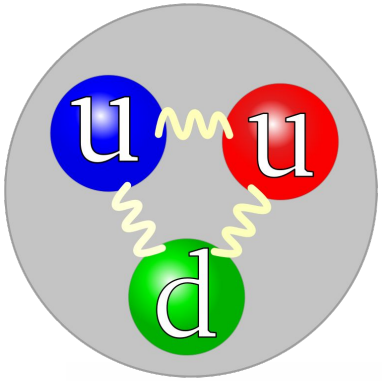
Color Charge



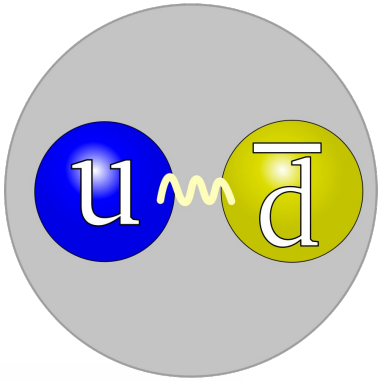


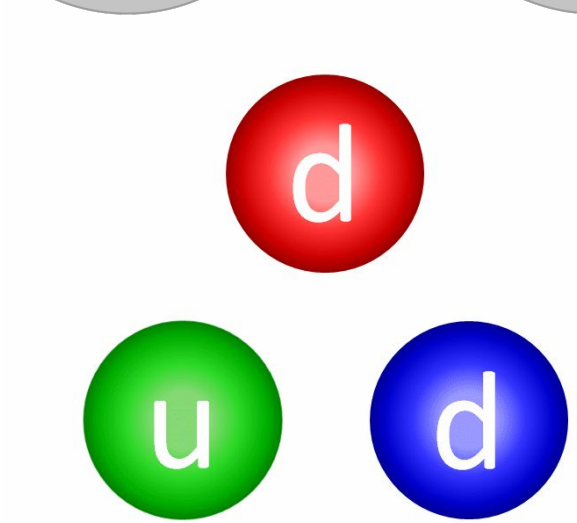


Proton

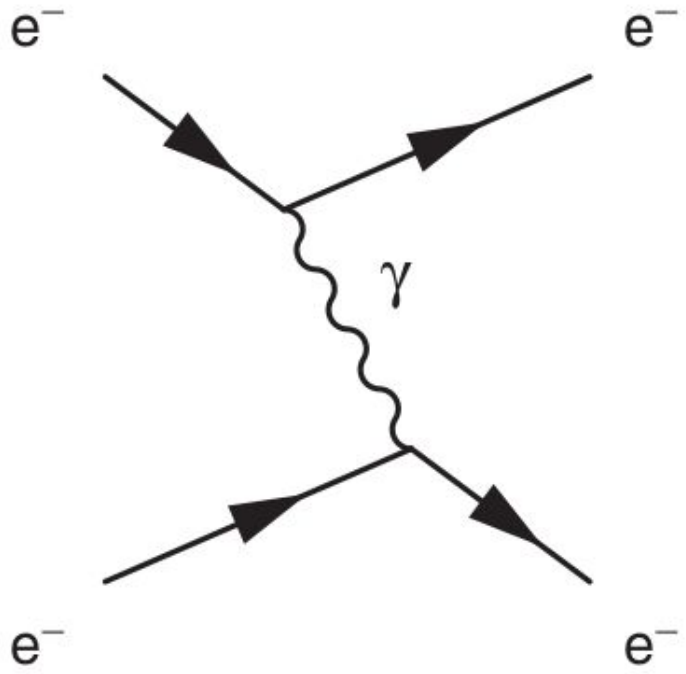


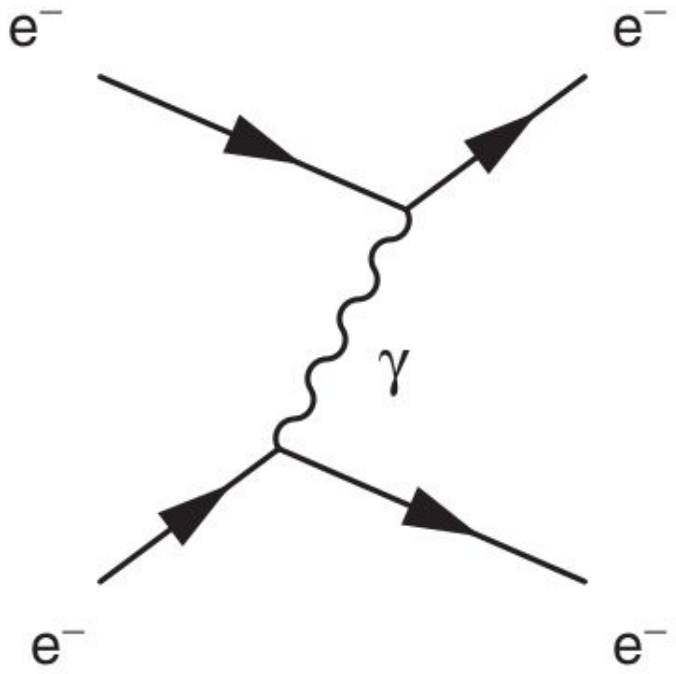
Pion



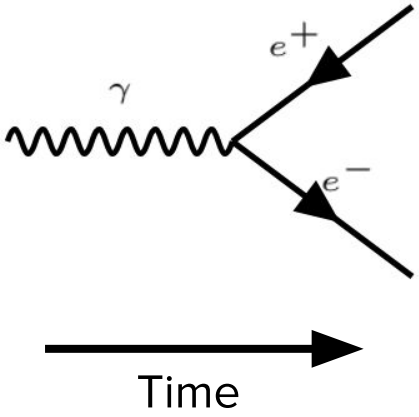


Feynman Diagrams

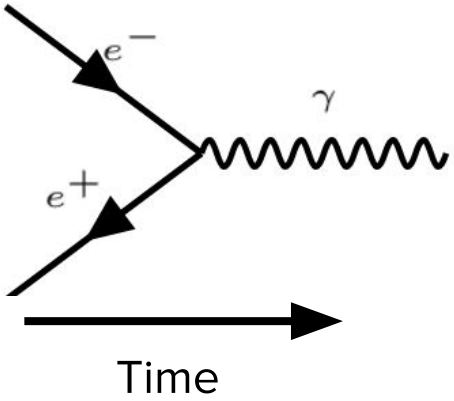




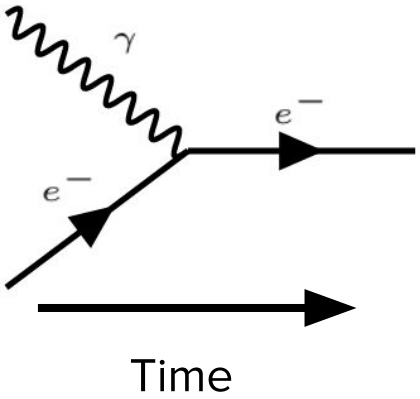
Electron Positron Creation



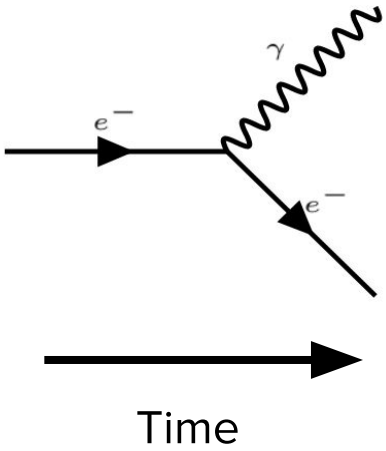
Electron Positron Annihilation



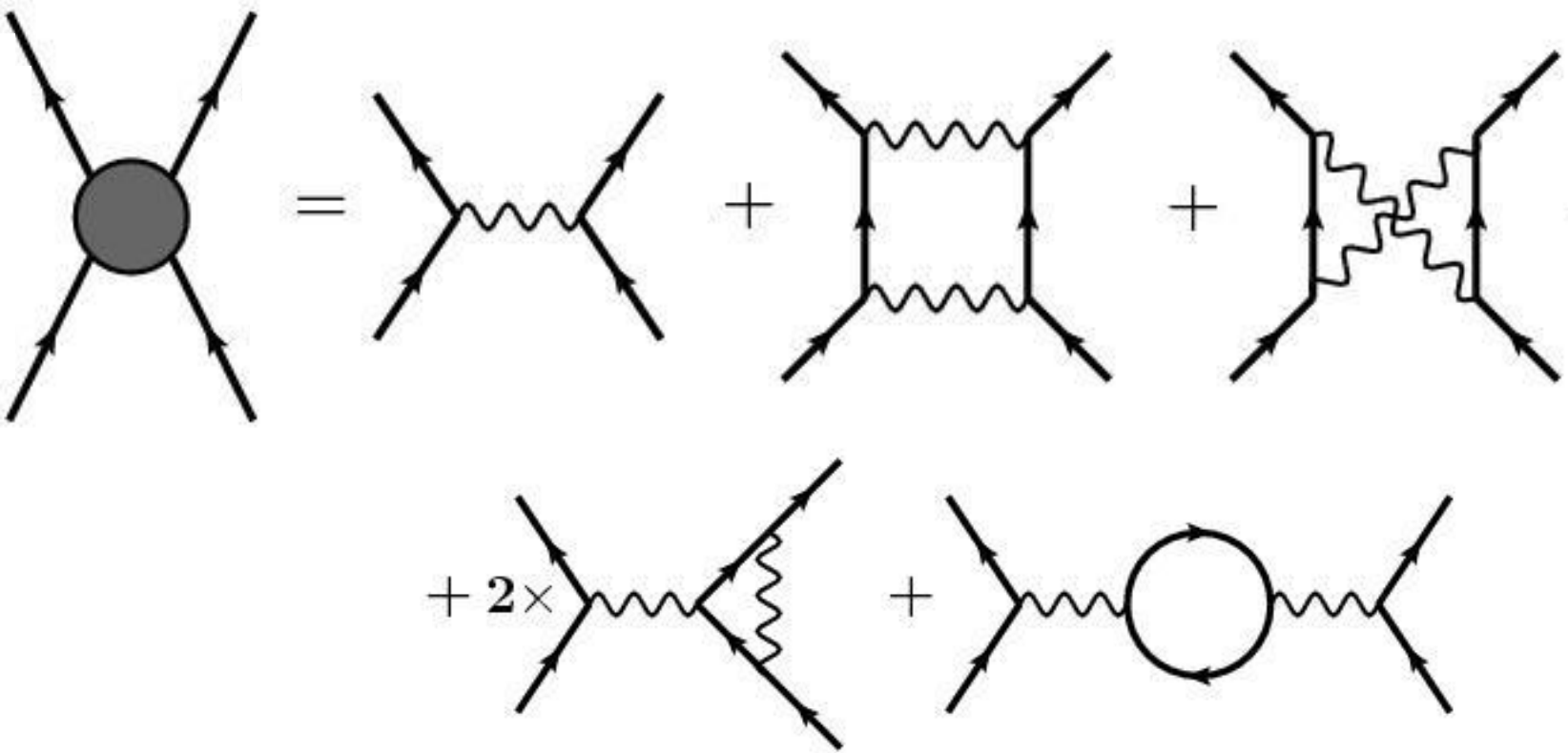
Photon Absorption



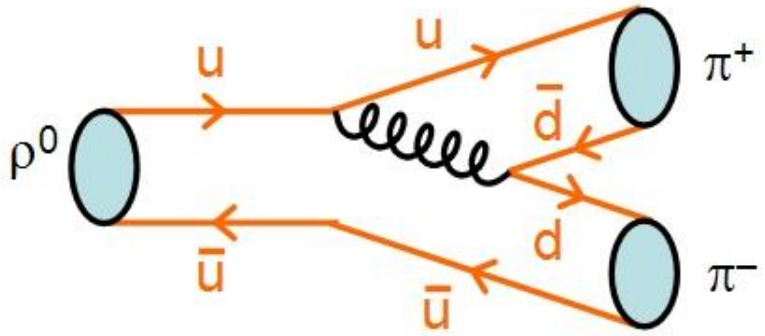
Photon Emission



Perturbation Theory

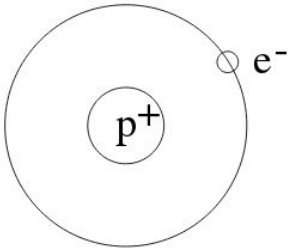


Quantum ChromoDynamics (QCD)



$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} - m\,\delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$

QED



Hydrogen Atom

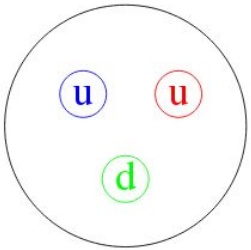
$M_e = 0.5$ MeV

$M_p = 938$ MeV

$E_{\mathrm{binding}} = 13.6$ eV

(EM force)

QCD



Proton

$M_u \sim 3$ MeV

$M_d \sim 6$ MeV

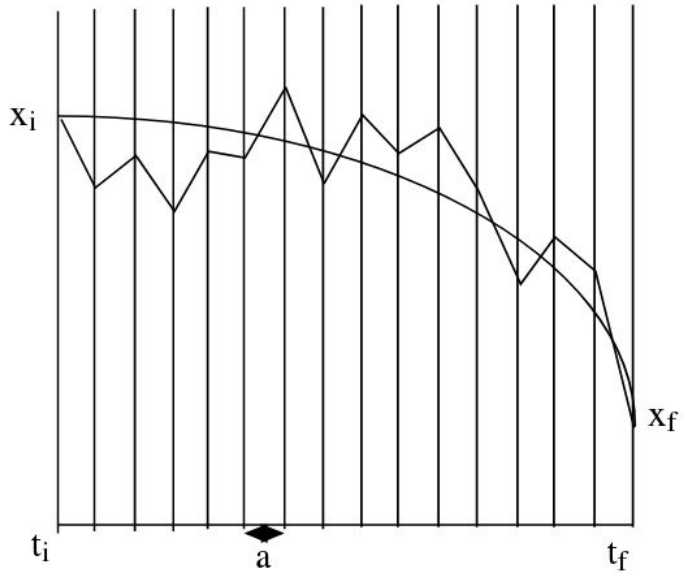
$M_p = 938$ MeV

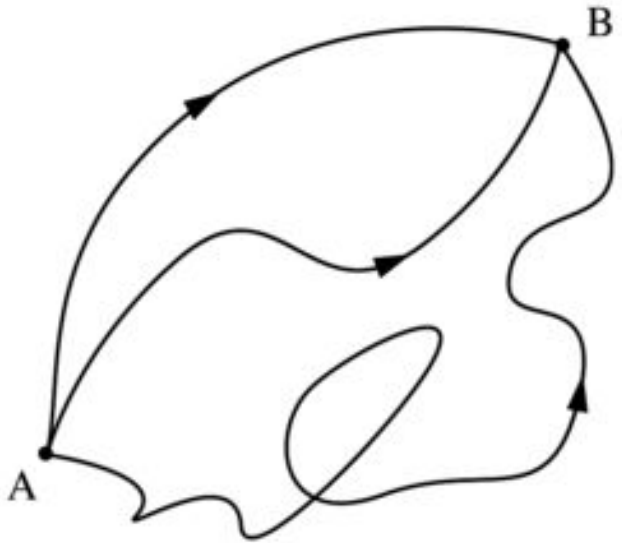
(Strong force)

Path Integral

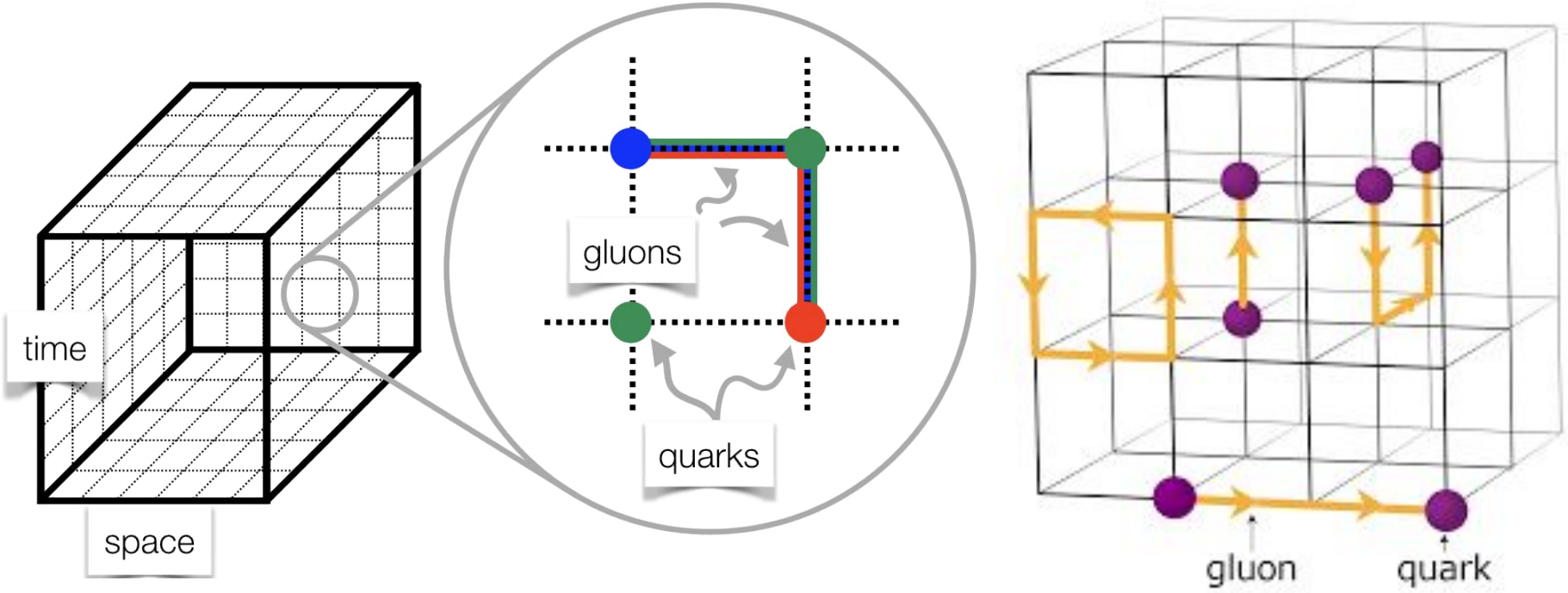
$$P_{i\to f} \sim \langle x_f(t_f)|x_i(t_i)\rangle = \int \mathcal{D}x(t)e^{iS[x]}$$

$$S[x] \equiv \int_{t_i}^{t_f} dtL(x,\dot{x}) \equiv \int dt[\frac{m\dot{x}(t)^2}{2} - V(x(t))].$$



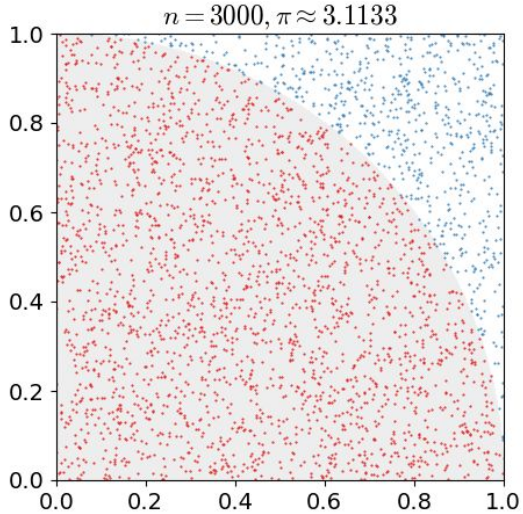


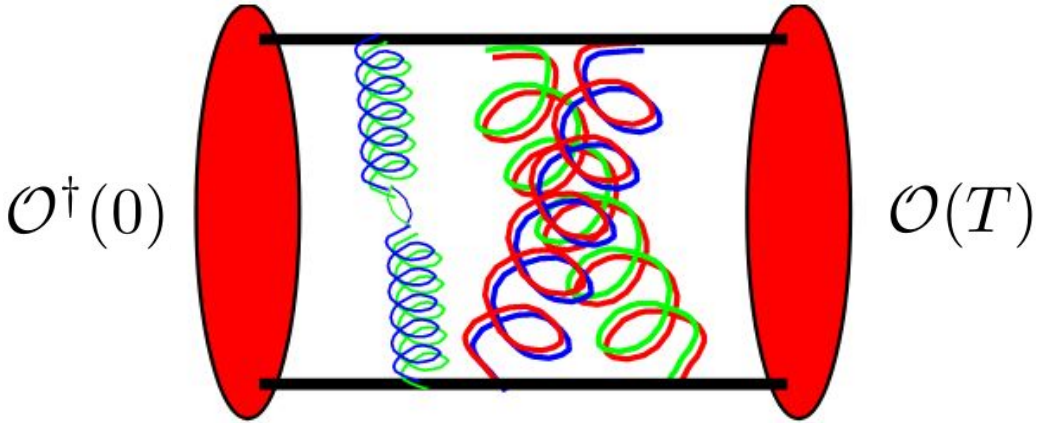
Lattice QCD



Monte Carlo

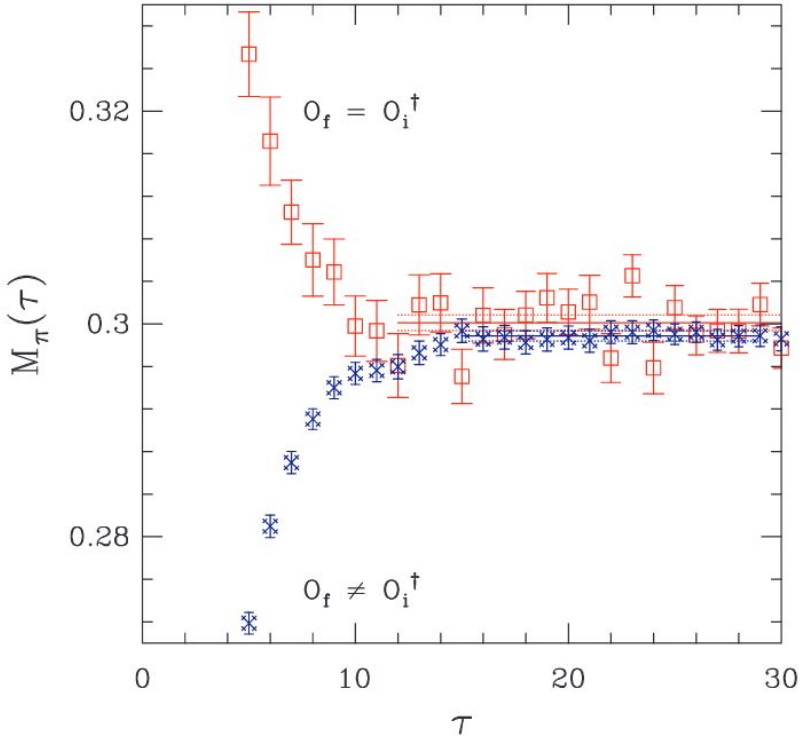
$$\frac{1}{Z} < 0|\mathcal{O}|0 >= \frac{\int \mathcal{D}U \mathcal{O}[U] e^{-S_{g,QCD}}}{\int \mathcal{D}U e^{-S_{g,QCD}}} =<< \mathcal{O} >>= \frac{1}{N_{conf}} \sum_{i=1}^{N_{conf}} O_i$$

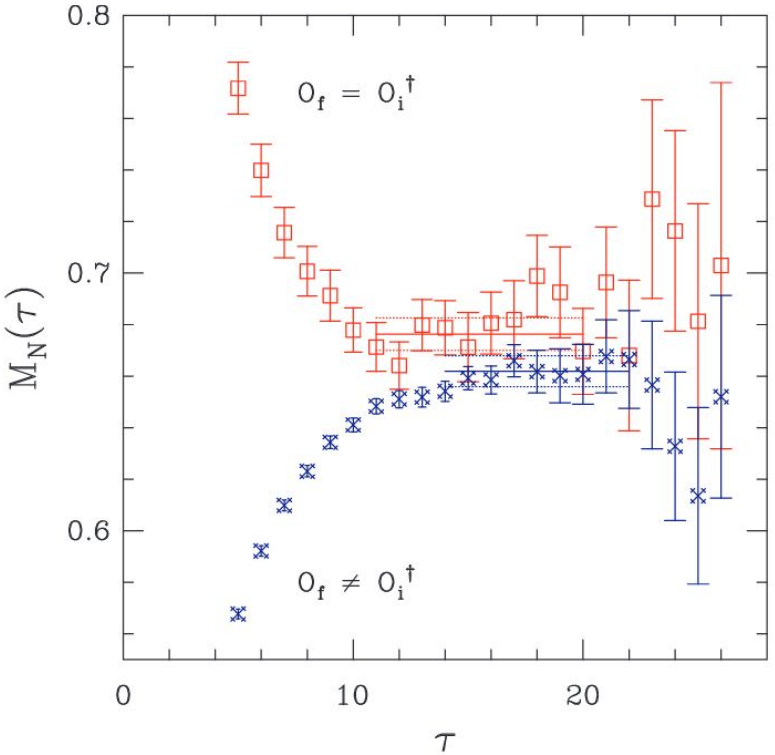


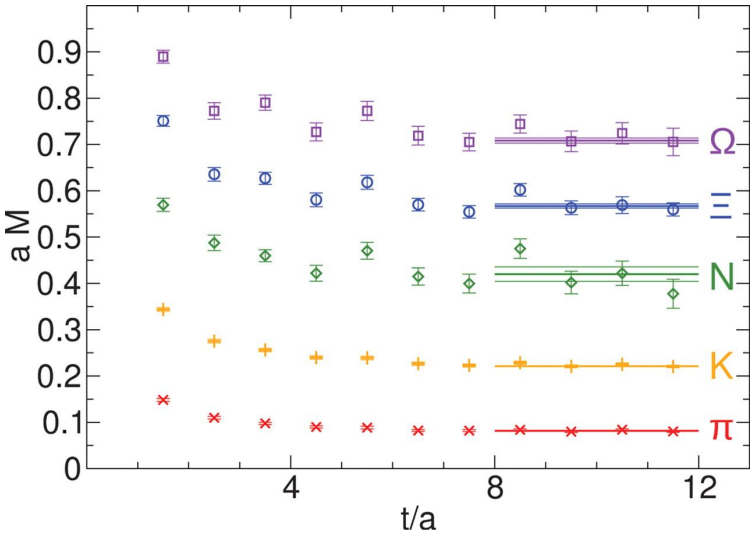


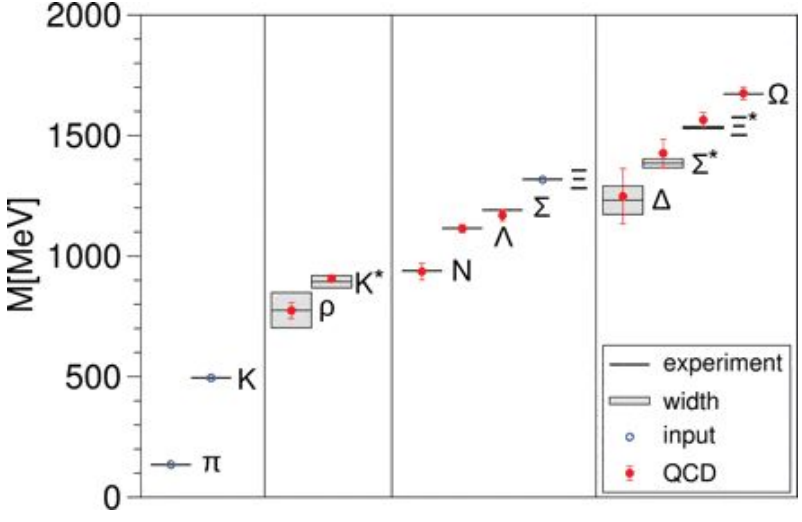
Energies

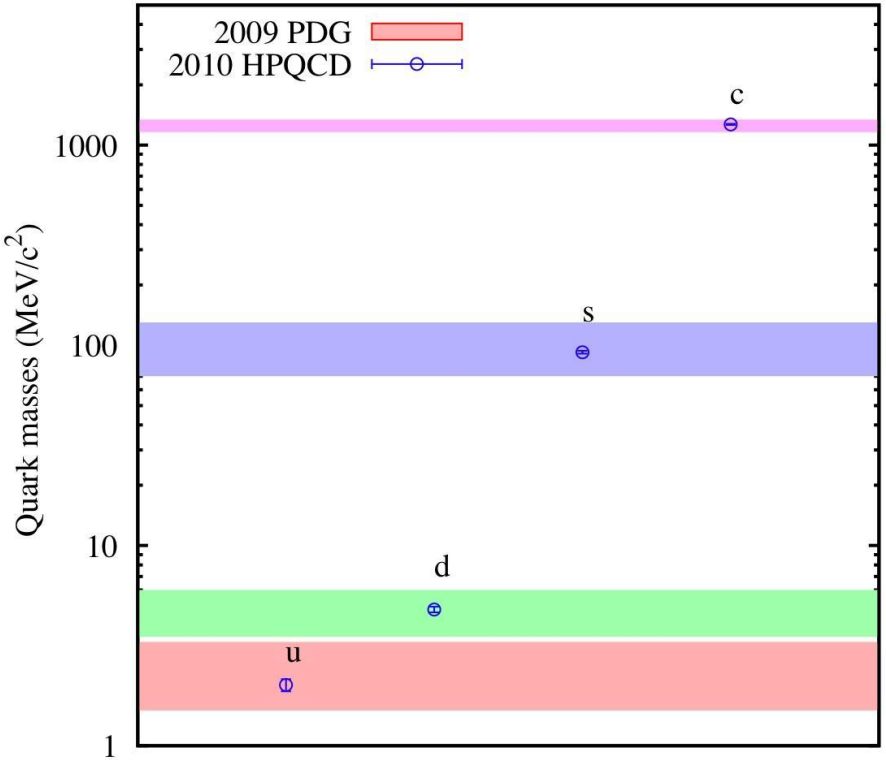
$$P_{0\to T} \sim \langle 0|\mathcal{O}(T)\mathcal{O}^\dagger(0)|0\rangle = \sum_{n=0}^{\infty} c_n e^{-E_n T} \xrightarrow{T\to\infty} c_0 e^{-mT}$$






a M
t/a
Ω
Ξ
N
K
π


M[MeV]
π
K
ρ
K*
N
Λ
Σ
Ξ
Δ
Σ*
Ξ*
Ω
experiment
width
input
QCD


2009 PDG
2010 HPQCD
Quark masses (MeV/c²)
c
s
d
u

More applications

- Decay constants
- Resonances
- Investigate color confinement
- Deep inelastic scattering
- High temperatures
- QCD phase transitions